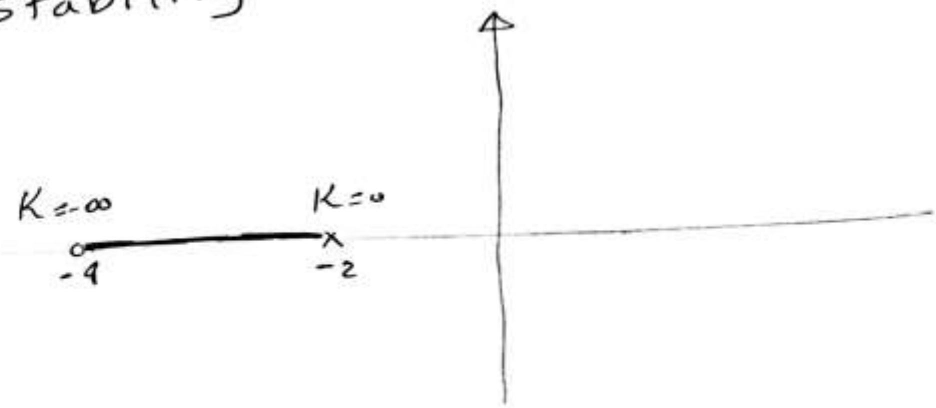
# Control Lec 3

**quiz** $GH(s) = \dfrac{K(s+4)}{(s+2)^3}$

→ Draw Root locus . find range of K for stability

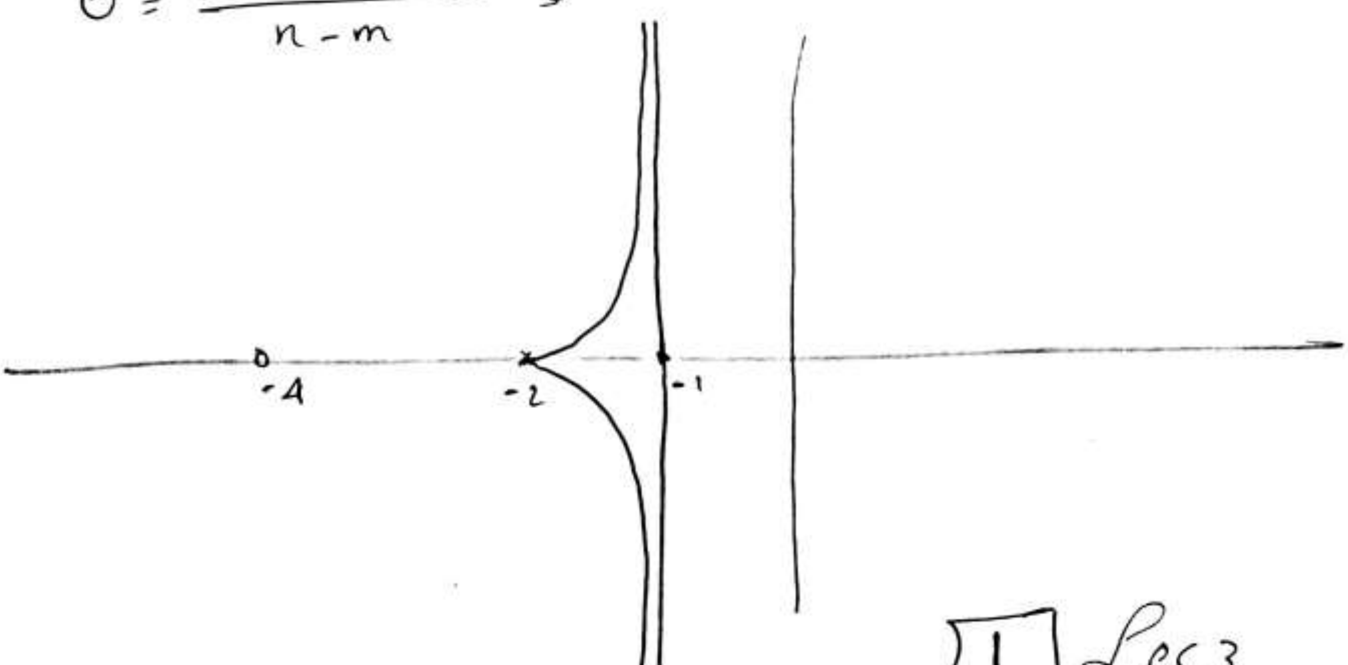



## Asymptotes

number $= 3 - 1 = 2$

$$C_A = \frac{(-2-2-2) - (-4)}{-2} = -1$$

$$\theta = \frac{(2L+1)180}{n-m} \Rightarrow \theta_1 = 90^\circ \; / \; \theta_2 = -90^\circ$$




$\boxed{1}$ Lec3

$\boxed{Ex}$

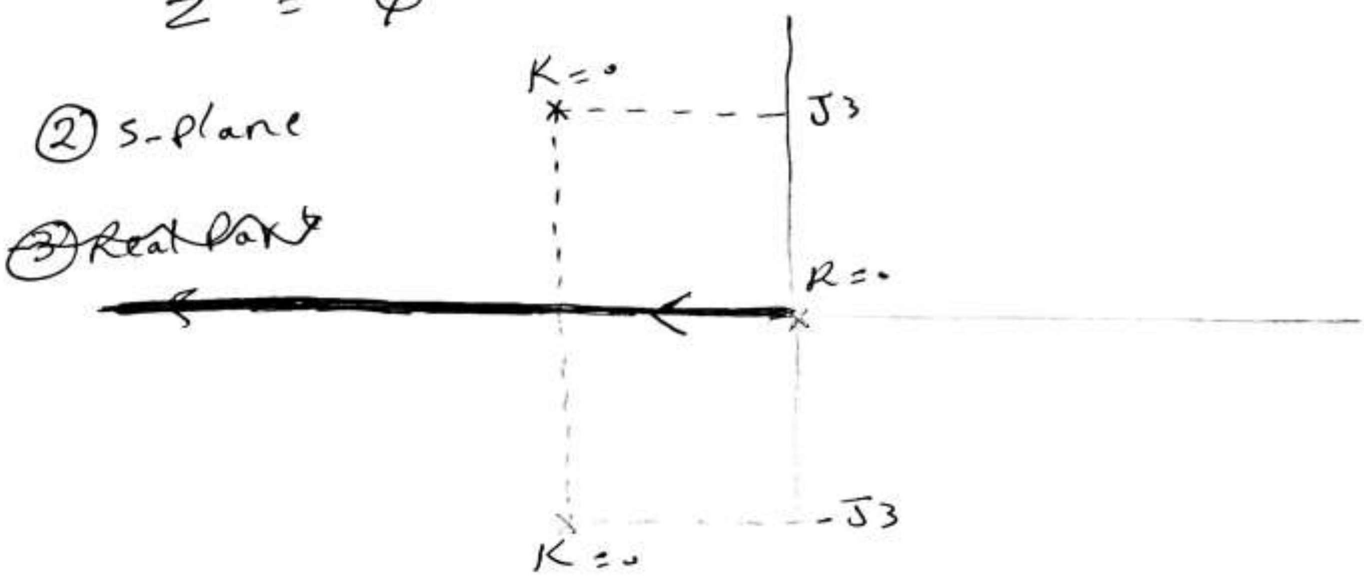
$$GH(s) = \frac{K}{s(s^2+4s+13)}$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16-4*13}}{2}$$

① Poles ⇒ $0, -2+J3, -2-J3$

$Z = \phi$

② s-plane




③ Real Part ⇒ $0 \longrightarrow -\infty$

$\boxed{4}$ Asymptotes

→ number $= 3-0 = 3$

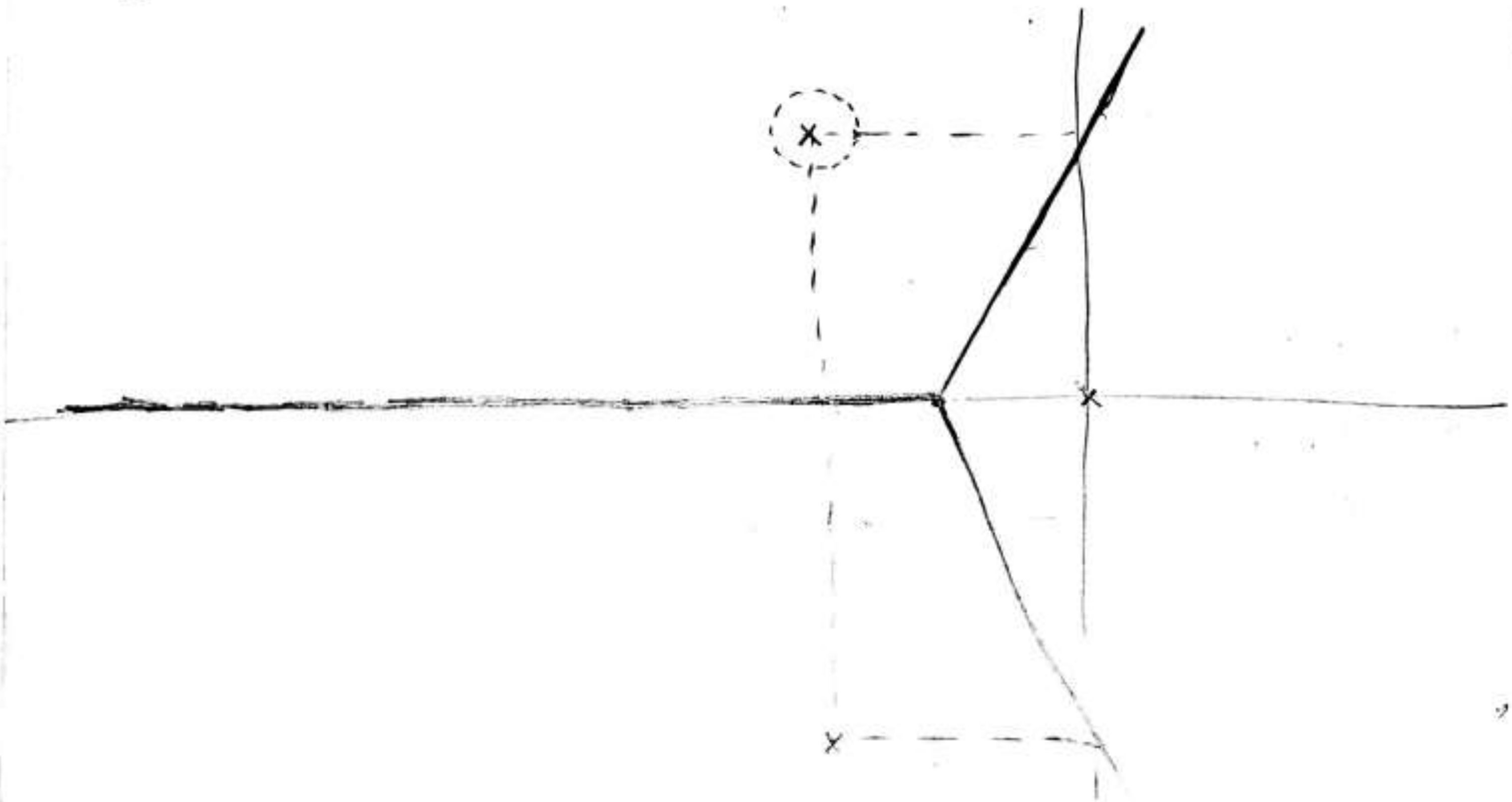
$$\rightarrow C_A = \frac{(0+J3-2-2-J3)-(0)}{3} = \frac{-4}{3} = -1{,}33$$

$$\theta = \frac{(2L+1)\,180}{3}$$

$L = 0 \Rightarrow \theta_1 = 60$

$L = 1 \Rightarrow \theta_2 = 180$

$L = 2 \Rightarrow \theta_3 = 300 = -60^\circ$

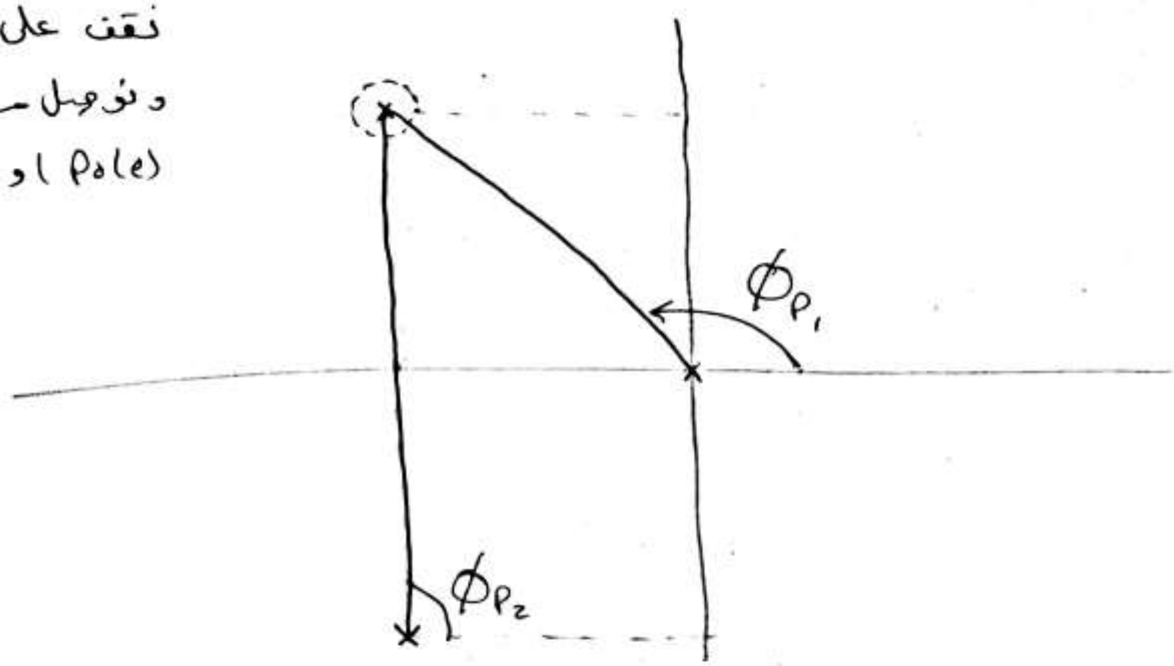


⑤ Departure Angle

$$\theta_D = 180 - \phi_P + \phi_Z$$

مجموع زوایا

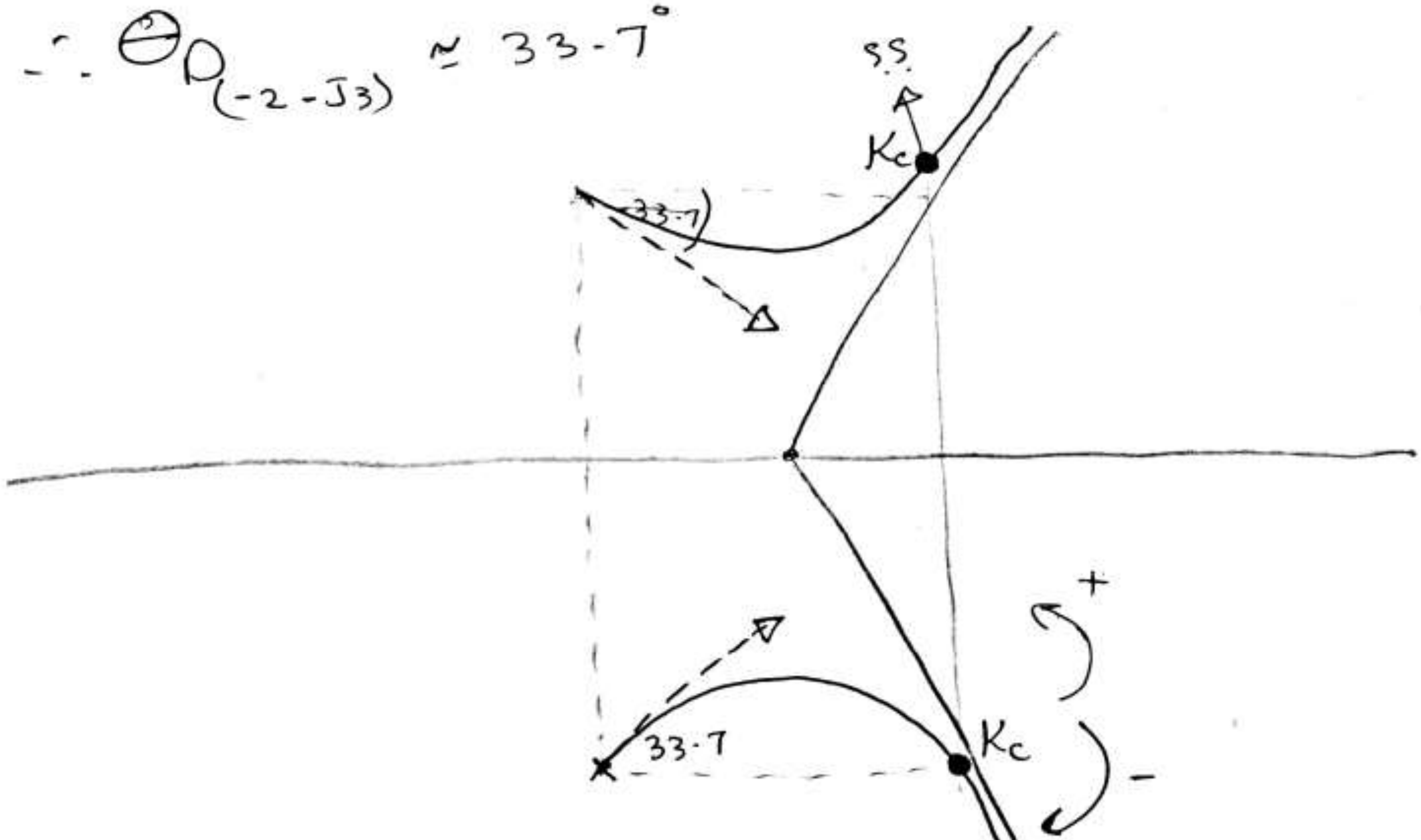
نقف على كل (Pole)
ونوصل منه خط لكل
(Pole) او (zero)




$$\theta_{D_{(-2+J3)}} = 180 - (\phi_{P_1} + \phi_{P_2}) + 0$$

$$= 180 - \left[\left(180 - \tan^{-1}\left(\tfrac{3}{2}\right)\right) + 90\right] \simeq -33.7^\circ$$

$$\therefore \theta_{D_{(-2-J3)}} \simeq 33.7^\circ$$



$\boxed{6}$ <u>At imag. Axis</u>

ch. eq $\Rightarrow$ $1 + GH(s) = 0$

$$1 + \frac{K}{s^3 + 4s^2 + 13s} = 0$$

$$s^3 + 4s^2 + 13s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 13 \\ s^2 & 4 & K \\ s^1 & \frac{52-K}{4} & > 0 \rightarrow (1) \\ s^0 & K & > 0 \rightarrow (2) \end{array}$$

$$\frac{52-K}{4} > 0$$

$$52 - K > 0$$

$$K < 50 \longrightarrow (1)$$

$$K > 0 \longrightarrow (2)$$

range of stability for K $\quad 0 < K < 52$

$K_{critical} = 52$ اللى بتخلى فيه جذور عالمحور الرأسى

$$A(s) = 4s^2 + K_c = 0$$

$$4s^2 + 52 = 0$$

$$\boxed{s = \pm j3.6}$$

$\boxed{5}$ Lec3

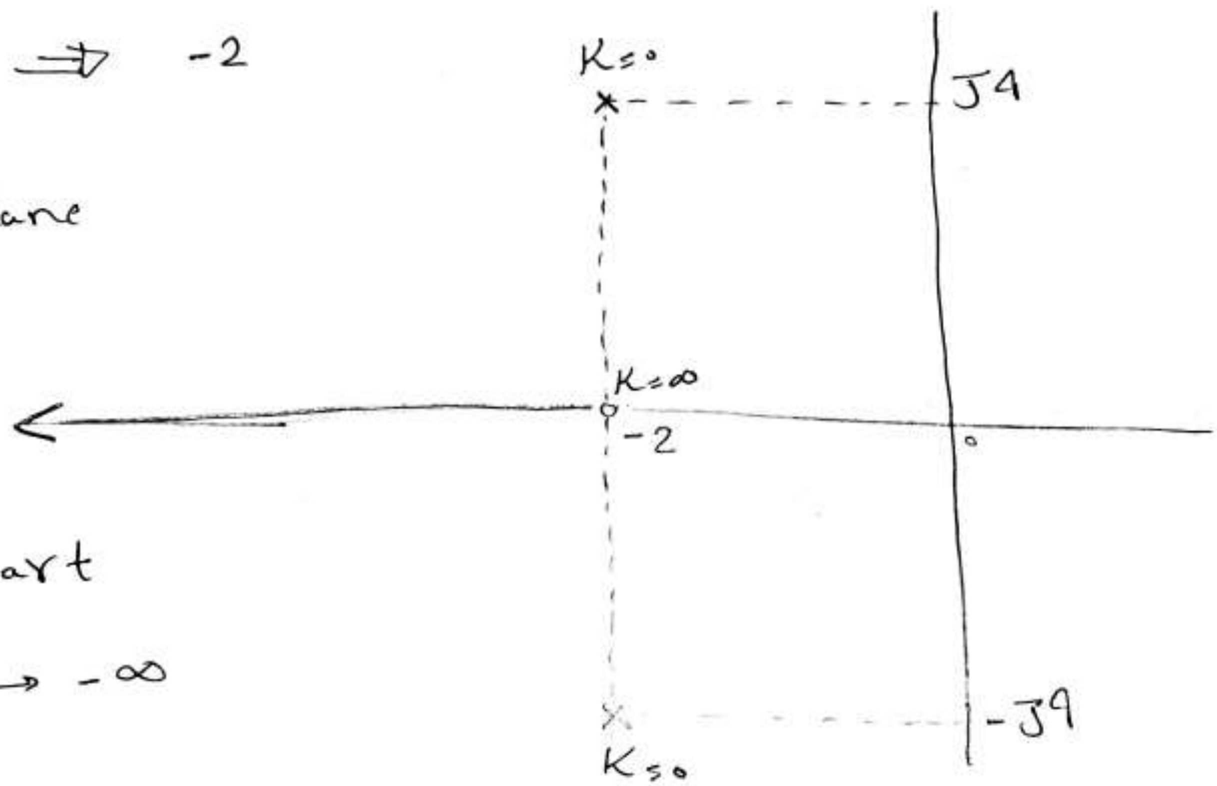
$\boxed{Ex}$

$$GH(s) = \frac{K(s+2)}{s^2+4s+20}$$

① Poles ⇒ $-2+J4$ & $-2-J4$

Zeros ⇒ $-2$

② s-Plane




③ real Part

$-2 \to -\infty$

## $\boxed{4}$ Breaking Point

ch. eqn $\quad 1 + GH(s) = 0$

$$GH(s) = -1$$

$$\frac{K(s+2)}{s^2+4s+20} = -1$$

$$K = -\left[\frac{s^2+4s+20}{s+2}\right] \longrightarrow *$$

$$\frac{dK}{ds} = 0 \Rightarrow -\frac{(s+2)(2s+4)-(s^2+4s+20)(1)}{(s+2)^2}$$

$$(s+2)(2s+4)-(s^2+4s+20) = 0$$

$$2s^2+8s+8-s^2-4s-20 = 0$$

$$s^2+4s-12 = 0$$

$$(s-2)(s+6) = 0$$

$s = 2$ xx or $s = -6$ ✓✓

in *

Breaking Point $(S_b) = -6$

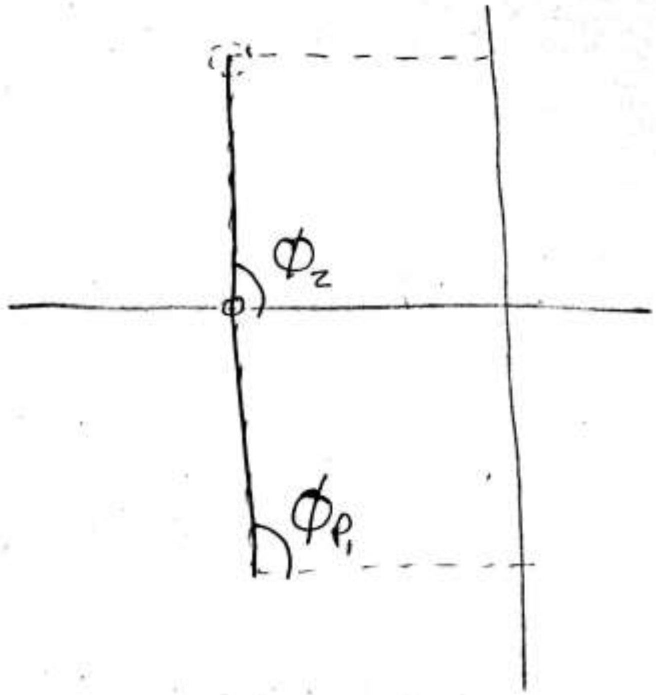
$$K\Big|_{s_b=-6} = 8$$

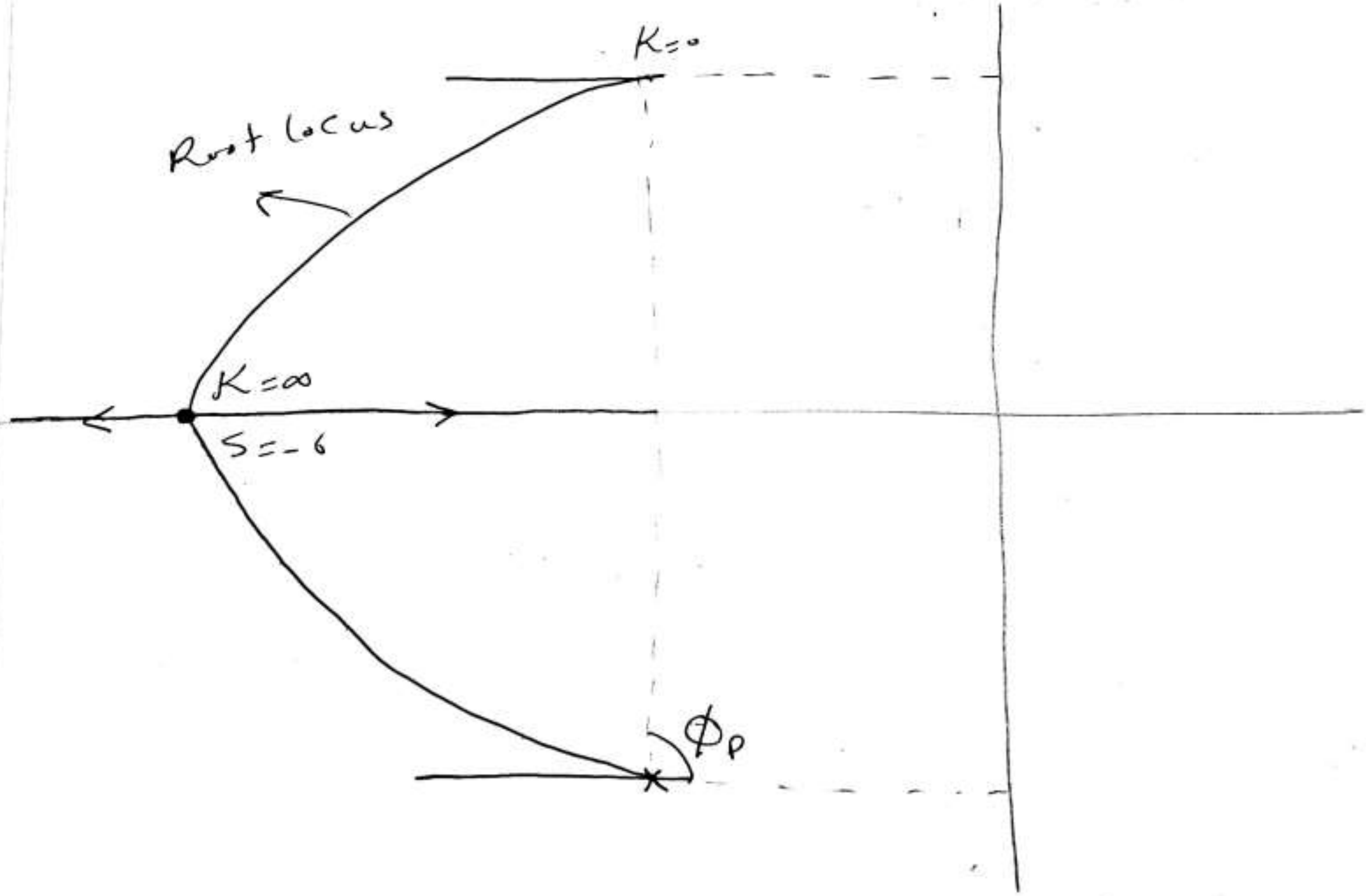
$\boxed{5}$ Dep. Angle

$$\phi_{0_{-2+J4}} = 180 - 90 + 90$$

$$= 180^\circ$$

$$\phi_{0_{-2-J4}} = -180^\circ$$




$\longrightarrow$ stable for all $K > 0$

$\boxed{Ex}$ $$GH(s) = \frac{20(1+Ks)}{s(s+1)(s+4)}$$

<u>ch-eqn</u> $1 + GH(s) = 0$ $s^2 + 5s + 4$

$$1 + \frac{20(1+Ks)}{s(s+1)(s+4)} = 0$$

$$s(s+1)(s+4) + 20 + 20Ks = 0$$

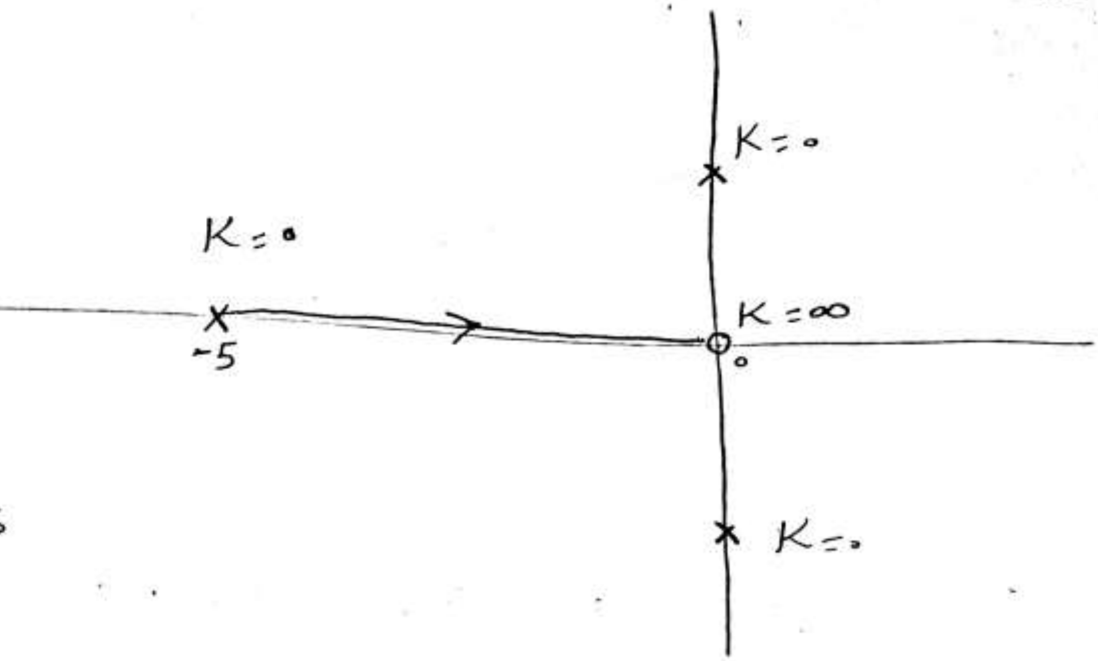
$$1 + \frac{20Ks}{s(s+1)(s+4)+20} = 0$$

$$1 + GHs = 0$$

$$GH(s) = \frac{20Ks}{s(s+1)(s+4)+20}$$

① Poles $\Rightarrow -5, J2, -J2$

Zeros $\Rightarrow$ 0




③ Real Part

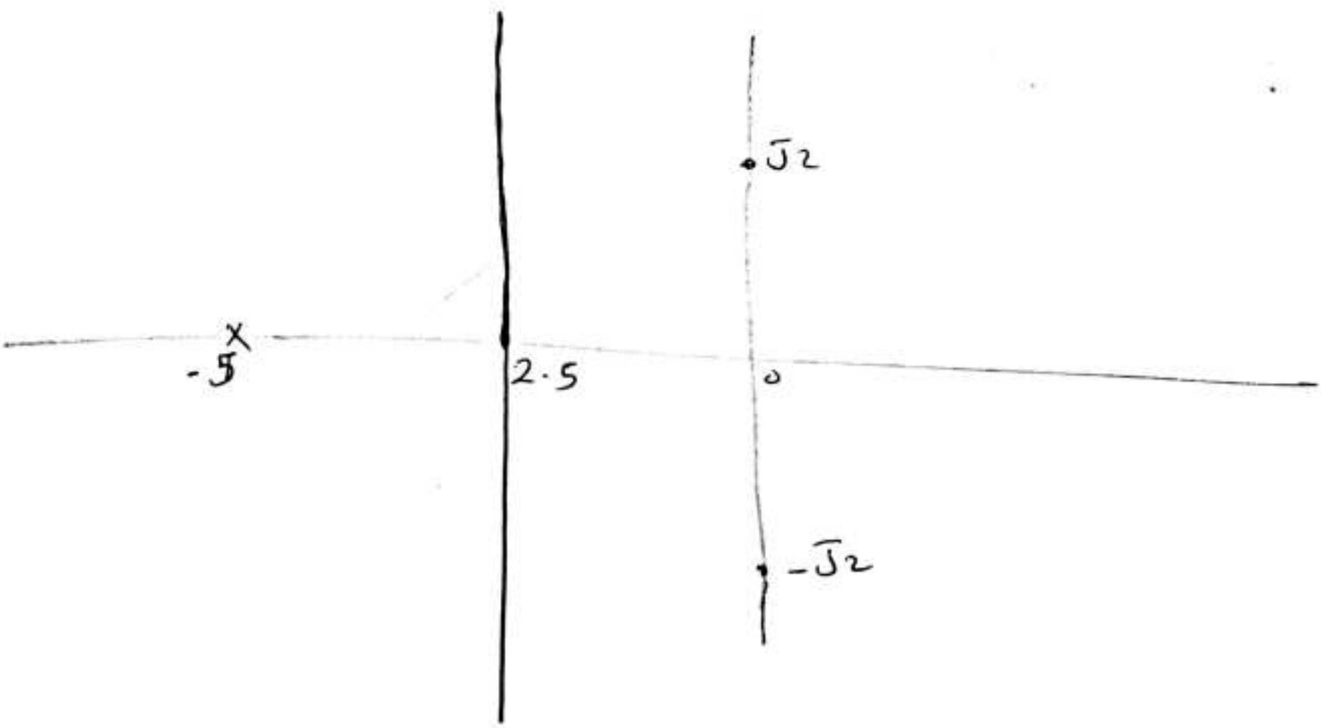
$0 \longrightarrow -5$

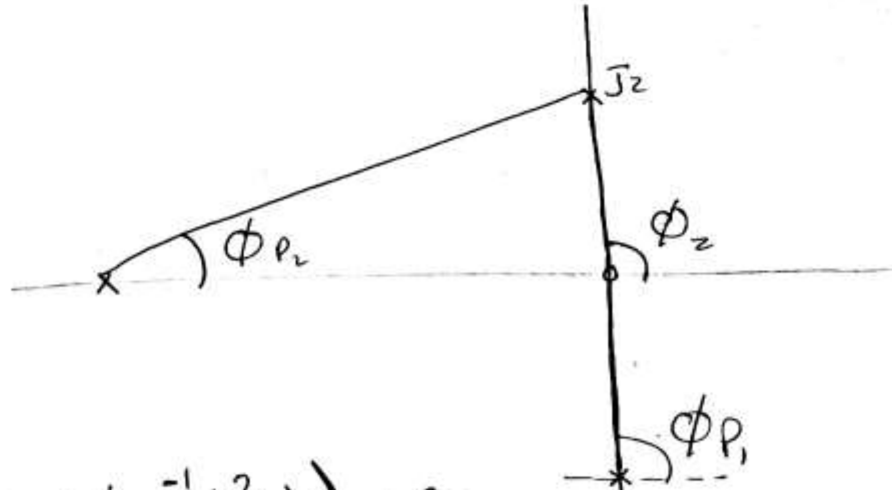
④ Asymptotes

① number $= 3 - 1 = 2$

② $C_A = \dfrac{(-5 + J2 - J2) - 0}{2} = \dfrac{-5}{2} = -2.5$

$\theta = \dfrac{(2L+1)\,180}{2} = 90, -90$



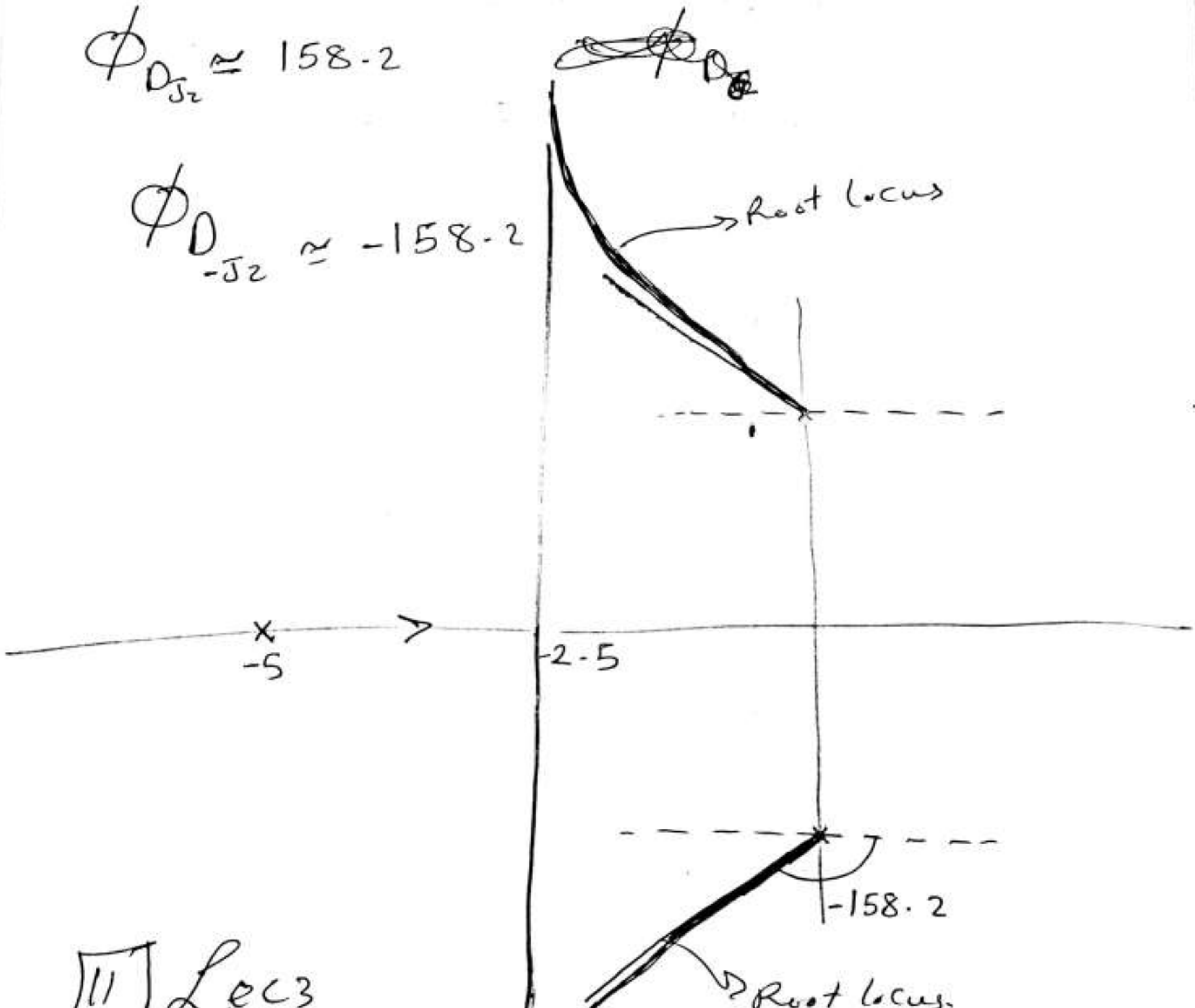



$\boxed{5}$ Def. Angle

$$\phi_{D_{j2}} = 180 - (\underbrace{90}_{\phi_{P_1}} + \underbrace{\tan^{-1}\left(\tfrac{2}{5}\right)}_{\phi_{P_2}}) + \underbrace{90}_{\phi_z}$$

$$\phi_{D_{j2}} \simeq 158.2$$

$$\phi_{D_{-j2}} \simeq -158.2$$



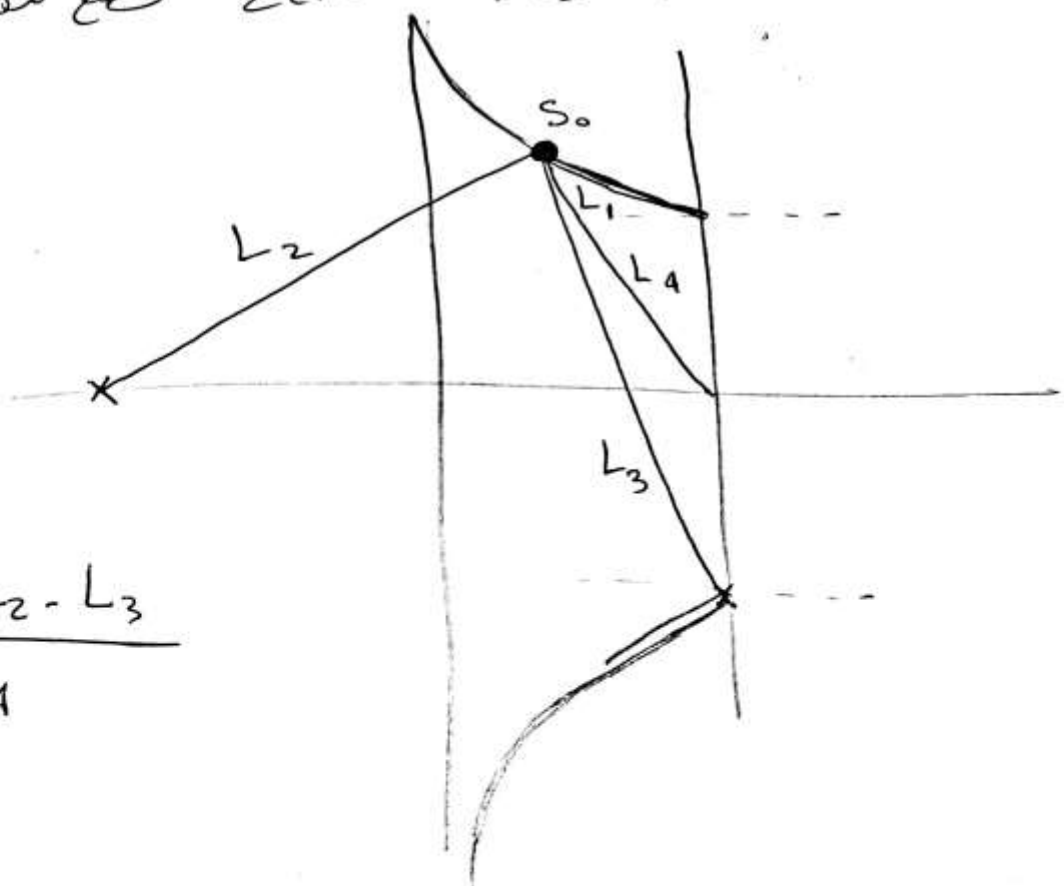

$\boxed{11}$ Lec3

## $\boxed{1}$ Find K at $S_o$

$S_o \leftarrow$ نقطة تقع على ال (root locus)

$$K\Big|_{S_o} = \frac{\Pi \text{ Poles}}{\Pi \text{ Zero}}$$

← في حالة إذا يوجد zero نضع مكانها 1.




$$K\Big|_{S_o} = \frac{L_1 \cdot L_2 \cdot L_3}{L_4}$$

## $\boxed{2}$ K at $\zeta = 0.5$

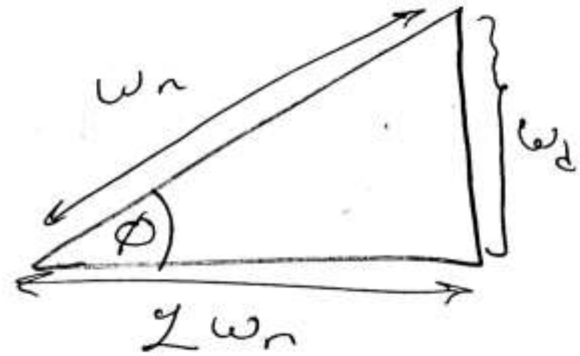
$\zeta \rightarrow$ damping ratior

$$T.F = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n + \omega_n^2}$$

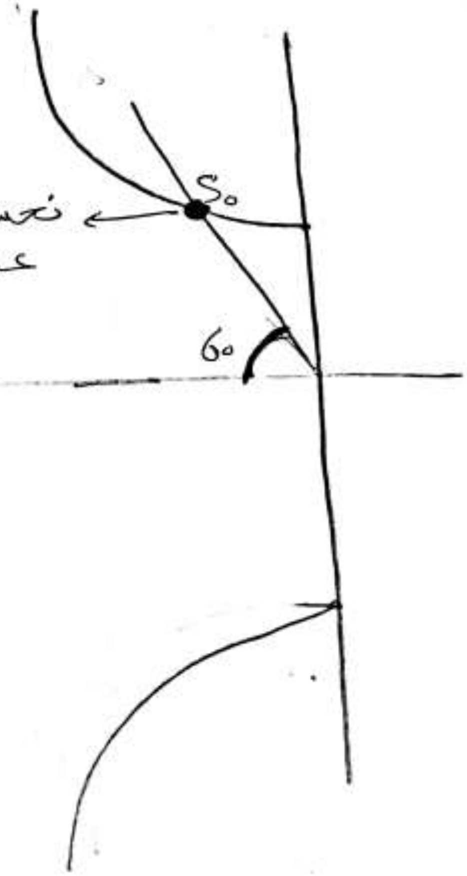
$$S_{1,2} = -\zeta\omega_n \pm J\omega_d$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\phi = \cos^{-1}\zeta = 60^\circ$$




← بنقيس زاوية مع المحور الأفقى بالسالب




← نحسب K عند النقطة دي.

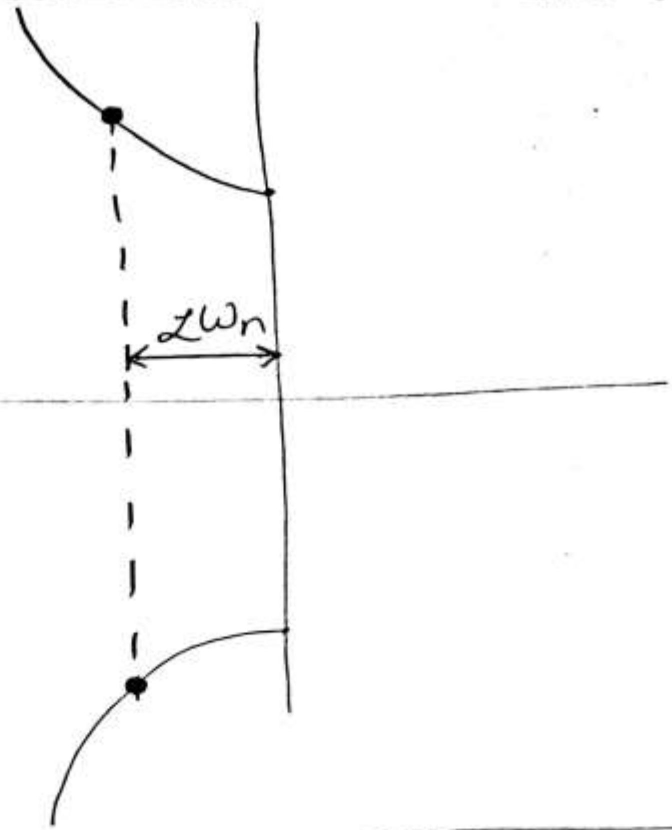
← لو عملت الزاوية من تحت هتطلع نفس الشكل.

$\boxed{3}$ find $K$ at $t_s = 2$ sec

~~$t_s$~~ given $\zeta\omega_n$

$$t_s = \frac{4}{\zeta\omega_n} \quad (2\%\ \text{error})$$

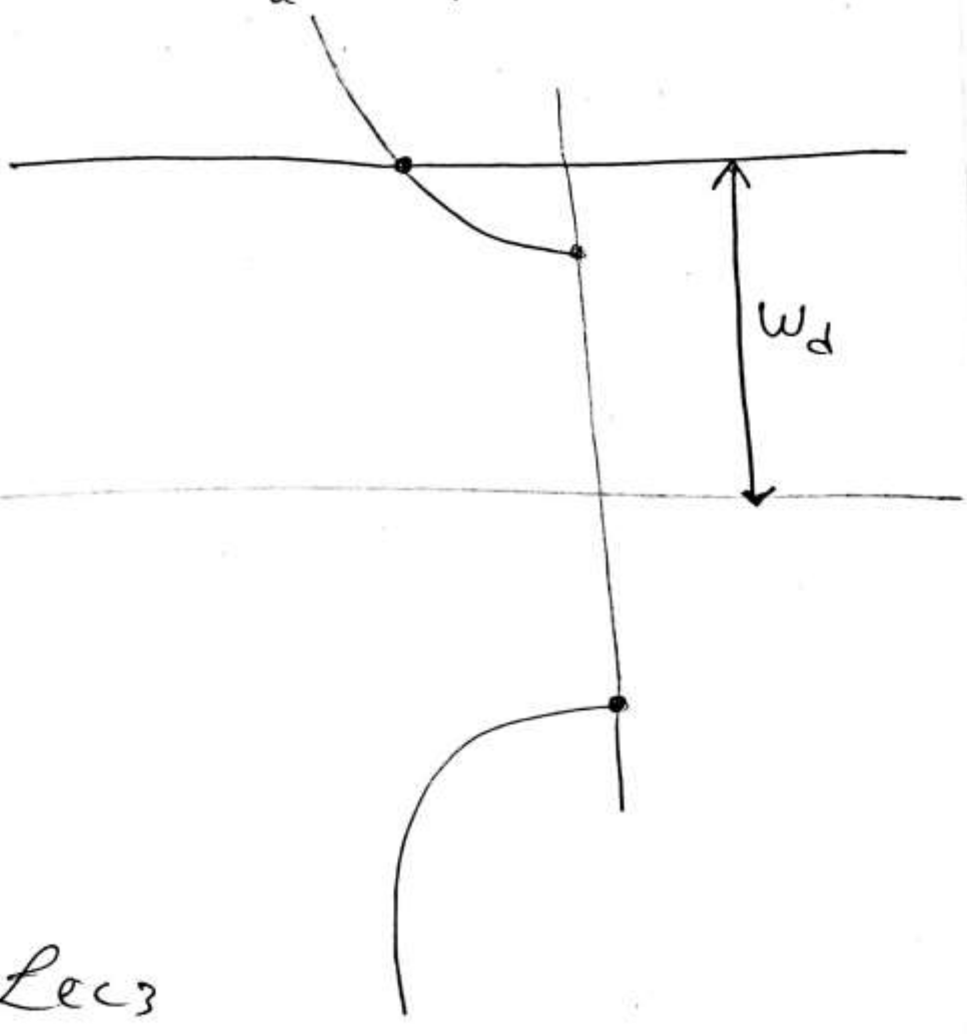
ـــ خد نقطة منهم ودوصل
خطوط منها للـ pole ، zero
واحسب K كما في رقم (1)




$\boxed{4}$ K at $\omega_d = \ldots$

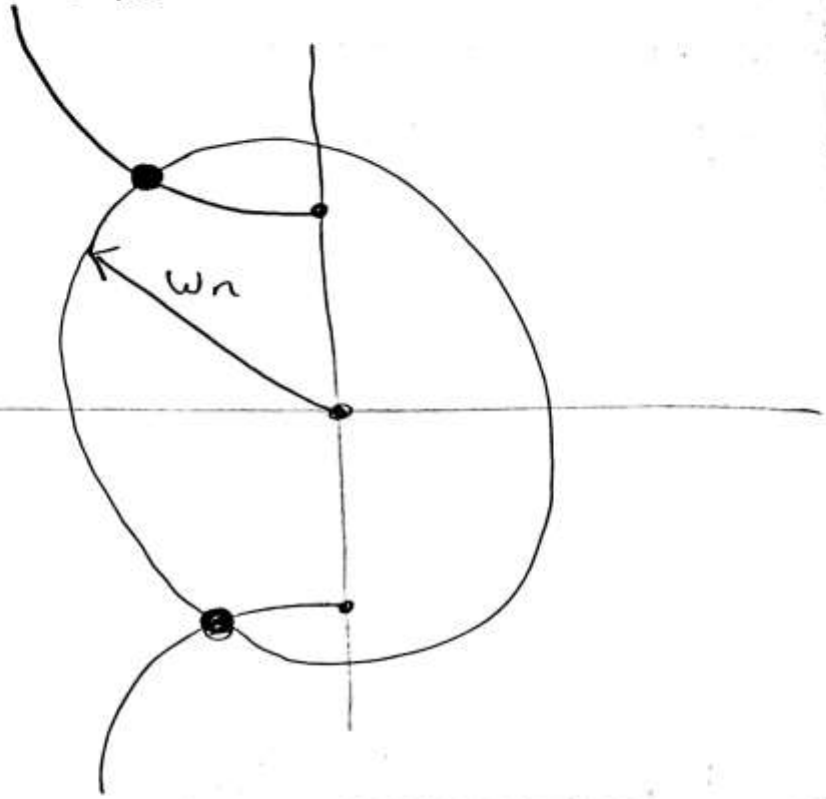
↳ damping frequency (rad/sec)

إبعد عن الأفق مسافة
($\omega_d$) لأعلى أو لأسفل
وارسم خط أفقي هيتقاطع
مع الـ (root locus)



$\boxed{5}$ Find K at $\omega_n$ = rad/sec

ـ عند نقطة الأصل
ترسم دائرة نصف قطرها
يساوي (Wn) ستتقاطع
مع ال (root locus)

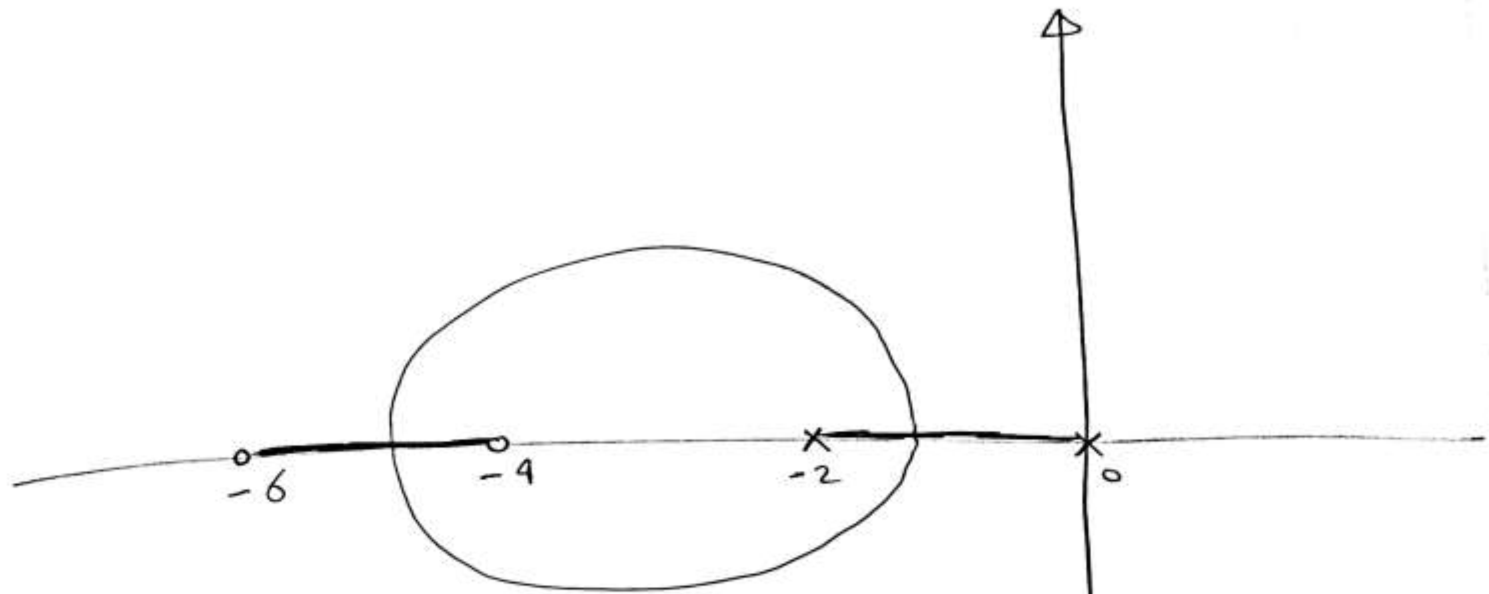



Report

$$GH(s) = \frac{K(s+4)(s+6)}{s(s+2)}$$

- Draw root locus
- Find the range of K for stability.
- K at $t_s = 2\ sec = \frac{4}{\zeta\omega_n} \Rightarrow \zeta\omega_n = 2$
- K at $\zeta = \frac{\sqrt{3}}{2} \Rightarrow \phi = \cos^{-1}\zeta = 30^\circ$

← على ورقة رسم بياني.




**Real Part**

$0 \rightarrow -2$

$-4 \rightarrow -6$

**Breaking point** هتطلعلك دائرة

← إرسم الدائرة على ورقة رسم بياني .

← لو كان شكل المثال

$$GH(s) = \frac{10K\,(S+4)(S+6)}{S(S+2)}$$

Put $10K = K'$